

# Solutions

## Chapter 11: Project: The game of Nim

### Six coins

When we start with six coins we should take one coin to win because:

- If we take one coin, we leave our opponent with five – which we showed was a losing position for them.
- If we take two coins (leaving four), our opponent will take three – we lose.
- If we take three coins (leaving three), our opponent will take two – we lose.

Therefore, the winning strategy is to take one coin, leaving our opponent with the losing position of five coins.

### Seven coins

When we start with seven coins we should take two coins to win because:

- If we take one coin (leaving six), our opponent will take one – leaving us with five – we lose.
- If we take two coins, we leave our opponent with five – they lose.
- If we take three coins (leaving four), our opponent will take three – we lose.

Therefore, the winning strategy is to take two coins, leaving our opponent with the losing position of five coins.

### Eight coins

When we start with eight coins we should take three coins to win because:

- If we take one coin (leaving seven), our opponent will take two – leaving us with five – we lose.
- If we take two coins (leaving six), our opponent will take one – leaving us with five – we lose.
- If we take three coins, we leave our opponent with five – they lose.

Therefore, the winning strategy is to take three coins, leaving our opponent with the losing position of five coins.

### Nine coins

Never start with nine coins! This is because:

- If we take one coin (leaving eight), our opponent will take three – leaving us with five – we lose.
- If we take two coins (leaving seven), our opponent will take two – leaving us with five – we lose.
- If we take three coins (leaving six), our opponent will take one – leaving us with five – we lose.

Therefore, when we start with nine coins we will lose no matter what we do.

### 10 coins

When we start with 10 coins we should take one coin to win because:

- If we take one coin, we leave our opponent with nine – they lose
- If we take two coins (leaving eight), our opponent will take three – we lose.
- If we take three coins (leaving seven), our opponent will take two – we lose.

Therefore, the winning strategy is to take one coin, leaving our opponent with the losing position of nine coins.

## Best strategies to take

**Table 11.7**

| Number of coins in pile | We should take: | We would leave our opponent: | Do we win or lose?          |
|-------------------------|-----------------|------------------------------|-----------------------------|
| 1                       | 1               | 0                            | lose                        |
| 2                       | 1               | 1                            | win                         |
| 3                       | 2               | 1                            | win                         |
| 4                       | 3               | 1                            | win                         |
| 5                       | X               | X                            | lose no matter what we take |
| 6                       | 1               | 5                            | win                         |
| 7                       | 2               | 5                            | win                         |
| 8                       | 3               | 5                            | win                         |
| 9                       | X               | X                            | lose no matter what we take |
| 10                      | 1               | 9                            | win                         |
| 11                      | 2               | 9                            | win                         |
| 12                      |                 |                              |                             |
| 13                      |                 |                              |                             |
| 14                      |                 |                              |                             |
| etc.                    |                 |                              |                             |

### Can you complete this rule?

If our opponent takes  $N$  coins, we should always take  $4-N$ . If we go first we will always win using this strategy, unless we are forced to go first when the coin count is every fourth number – 1, 5, 9, 13, ... We want to leave our opponent with that many coins at the start of their turn. To do that, we must force the nearest losing number. If they take  $N$  coins, we take  $4-N$  to get to the next losing number.

For example, suppose we start with 20 coins. The nearest losing number is 17, so we take 3. Suppose our opponent takes 2, leaving 15. We then take  $4-2 = 2$  to get to the next losing number, which is 13. Suppose they then take 3, leaving 10. We then take  $4-3 = 1$  to force then to face the next losing number, which is 9. Suppose they take 1, leaving 8. We then take  $4-1 = 3$  to get to the next losing number, which is 5. At this point, they realise they can't win, but take 1 as a formality, leaving us 4. We take 3, and they must take the last one – we win!